

In-loop squeezing is real squeezing to an in-loop atom

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Electro-optical feedback can produce an in-loop photocurrent with arbitrarily low noise. This is not regarded as evidence of ‘real’ squeezing because squeezed light cannot be extracted from the loop using a linear beam splitter. Here I show that illuminating an atom (which is a nonlinear optical element) with ‘in-loop’ squeezed light causes line-narrowing of one quadrature of the atom’s fluorescence. This has long been regarded as an effect which can only be produced by squeezing. Experiments on atoms using in-loop squeezing should be much easier than those with conventional sources of squeezed light.

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Squeezing is the reduction in noise in one quadrature of a light beam below the standard quantum limit at the expense of a corresponding increase in the noise of the conjugate quadrature [1]. The characteristic below and above shot-noise homodyne photocurrent spectra were first observed by Slusher *et al.* in 1985 [2]. Around the same time, the production of a below shot-noise amplitude-quadrature spectrum for a photocurrent which was part of a control loop was also first reported [3,4]. As pointed out by Shapiro *et al.* [9], this cannot be taken as evidence for squeezing in the conventional sense because the two-time commutation relations for an in-loop field are not those of a free field. Moreover, attempts to remove some of the supposedly low-noise light by a beam splitter yielded only above shot-noise light [3,5].

Very soon after the first observation of squeezing, Gardiner [6] made a seminal prediction regarding its effects on matter [7], namely that immersing an atom in broadband squeezed light would break the equality between the transverse decay rates for the two quadratures of the atomic dipole. In particular, one decay rate could be made arbitrarily small, producing an arbitrarily narrow line in the power spectrum of the atom’s fluorescence. This was seen, as the title of Ref. [6] proclaims, as a “direct effect of squeezing”.

In this letter I pose the question of whether this atomic line-narrowing is characteristic only of squeezing in the conventional sense (‘real squeezing’), or whether it can be produced by light which gives rise to a below shot-noise photocurrent by virtue of being part of a feedback loop (‘in-loop squeezing’). The answer is that in-loop squeezing *can* do the job. In fact, the dependence of the line-narrowing on the amount of squeezing and the quality of mode-matching to the atom are exactly the same for in-loop squeezing as for squeezing of a free field. Thus in-loop squeezing appears likely to be an important tool for future experimental investigation of the effect of low-noise light on atoms, as it is usually easier to generate than free squeezing.

The remainder of this letter is structured as follows. First I review the theory of in-loop squeezing. Then I

show how its effect on an in-loop atom can be described using quantum trajectories, including an arbitrary detector efficiency ε . In the limit of broad-band feedback I derive a master equation for the atom. From this the spectrum of the fluorescence of the atom into the other (non-squeezed) radiation modes is easily calculated. Then I compare these results with that obtained from broad-band free squeezing. Finally, I discuss experimental implications of the theory.

In-loop Squeezing. Consider the experimental apparatus shown in Fig. 1, but for the moment without the fluorescent atom. The Mach-Zehnder interferometer on the left hand side has two functions. First it produces a weak beam (b_{in} which is given by

$$b_{\text{in}}(t) = \nu(t) - (i/2)(\beta e^{i\phi} - \beta e^{-i\phi}) \approx \nu(t) + \beta\phi. \quad (1)$$

Here β , assumed real, is the coherent amplitude of the laser, and $\pm\phi$, assumed small, are the phase shifts imposed by the electro-optic modulators. The operator $\nu(t)$ represents vacuum fluctuations, for which all first and second order moments vanish except for [8]

$$\langle \nu(t)\nu^\dagger(t') \rangle = [\nu(t), \nu^\dagger(t')] = \delta(t - t'). \quad (2)$$

The second function of the interferometer is to produce a local oscillator beam with mean amplitude

$$-(1/2)(\beta e^{i\phi} + \beta e^{-i\phi}) = -\beta[1 + O(\phi^2)]. \quad (3)$$

With appropriate phase shifts assumed, this local oscillator is then used for making a homodyne measurement of the $X = b + b^\dagger$ quadrature of b_{out} (which, in the absence of the atom, is identical with b_{in}). If the efficiency of the detectors is ε then the homodyne photocurrent is represented by the operator [8]

$$I_{\text{hom}}(t) = \sqrt{\varepsilon} X_{\text{out}}(t) + \sqrt{1 - \varepsilon} \xi_\varepsilon(t), \quad (4)$$

where $\xi_\varepsilon(t)$ is a unit-norm real white noise process.

The photocurrent Eq. (4) is normalized so that the spectrum of the stochastic process $I_{\text{hom}}(t)$ (assumed to be stationary and of zero mean)

$$S_{\text{hom}}^X(\omega) = \langle \tilde{I}_{\text{hom}}(\omega) I_{\text{hom}}(0) \rangle \quad (5)$$

equals unity for $\omega \rightarrow \infty$. Here the tilde denotes the usual Fourier transform. The spectrum for X_{out} itself is defined analogously and is also given by

$$S_{\text{out}}^X(\omega) = \frac{1}{2\pi} \int d\omega' \langle \tilde{X}_{\text{out}}(\omega) \tilde{X}_{\text{out}}(-\omega') \rangle. \quad (6)$$

At very high frequencies $\tilde{X}_{\text{out}}(\omega)$ is dominated by the vacuum fluctuations $\tilde{\xi}_\nu(t) = \nu(t) + \nu^\dagger(t)$, which is a noise term like $\xi_\varepsilon(t)$. In the Fourier domain these obey

$$\langle \tilde{\xi}_a(\omega) \tilde{\xi}_b(-\omega') \rangle = 2\pi \delta_{a,b} \delta(\omega - \omega'), \quad (7)$$

as required for the limit $S_{\text{out}}^X(\infty) = S_{\text{hom}}^X(\infty) = 1$.

Although $S_{\text{hom}}^X(\omega)$ and $S_{\text{out}}^X(\omega)$ are shot-noise limited for high frequencies, they need not be for lower frequencies. In particular, the feedback loop shown can produce a spectrum below the shot-noise as follows. The current $I_{\text{hom}}(t)$ is amplified and used to control ϕ . If we set

$$\phi(t) = \frac{g}{2\beta\sqrt{\varepsilon}} \int_0^\tau h(s) I_{\text{hom}}(t-s) ds, \quad (8)$$

where $\tau > 0$ and $h(s)$ is normalized such that $\int_0^\tau h(s) ds = 1$, then we have a feedback loop with a low-frequency round-loop gain of g which is stable as long as $g\text{Re}[\tilde{h}(\omega)] < 1$ for all ω . Solving in the Fourier domain,

$$\tilde{X}_{\text{out}}(\omega) = \left[\tilde{\xi}_\nu(\omega) + g\tilde{h}(\omega) \sqrt{\frac{1-\varepsilon}{\varepsilon}} \tilde{\xi}_\varepsilon(\omega) \right] \frac{1}{1 - g\tilde{h}(\omega)}. \quad (9)$$

Thus X_{in} (which here equals X_{out}) has a spectrum

$$S_{\text{in}}^X(\omega) = [1 + g^2 |\tilde{h}(\omega)|^2 (\varepsilon^{-1} - 1)] / |1 - g\tilde{h}(\omega)|^2. \quad (10)$$

At a frequency $\bar{\omega}$ much less than the feedback bandwidth $\sim \tau^{-1}$, $\tilde{h}(\bar{\omega}) = 1$ and the minimum noise is

$$S_{\text{in}}^X(\bar{\omega})_{\text{min}} = 1 - \varepsilon, \quad \text{for } g = -\varepsilon/(1 - \varepsilon), \quad (11)$$

which is clearly below the standard quantum limit.

Note that the condition to minimize $S_{\text{in}}^X(\bar{\omega})$ is not the same as that to minimize the in-loop photocurrent noise:

$$S_{\text{hom}}^X(\bar{\omega})_{\text{min}} \rightarrow 0, \quad \text{as } g \rightarrow -\infty. \quad (12)$$

It might be thought that this is the more relevant quantity, since $S_{\text{in}}^X(\omega)$ is not actually measured in the experiment. However, a perfect QND [1] device for measuring X_{in} would produce the spectrum (10) [9]. It is thus expected that for an in-loop atom, the relevant spectrum would again be $S_{\text{in}}^X(\omega)$.

In-loop Atom. Returning to Fig. 1, we now include the two-level atom, which is assumed to be resonant to the laser. It couples strongly only to modes of the radiation

field having the appropriate dipole spatial distribution [8]. However, by focusing a beam as shown in Fig. 1, it is possible to mode-match a significant proportion, say η , of b_{in} into the atom's input. In practice, a more efficient way to increase the effective η would be to couple the light into a microcavity, as in Ref. [10]. The Hamiltonian of the atom in the interaction picture at time t is then

$$H(t) = -i[\sqrt{\eta} b_{\text{in}}(t) + \sqrt{1-\eta} \mu(t)] \sigma^\dagger(t) + \text{H.c.} \quad (13)$$

Here $\sigma = |g\rangle\langle e|$ is the atomic lowering operator and I have set the longitudinal atomic decay rate to unity. The operator $\mu(t)$ represents an independent vacuum input. Under this coupling, the output field is given by [8]

$$b_{\text{out}}(t) = b_{\text{in}}(t) + \sqrt{\eta} \sigma(t). \quad (14)$$

Although it would be possible to give a description of the entire feedback loop in terms of atomic and radiation field operators, it is simpler to use the quantum trajectory theory of homodyne measurement [11,12]. In this theory, only the atom is treated as a quantum mechanical system with state matrix $\rho(t)$; the rest of the apparatus is considered as a complicated measurement and feedback device for the atom. The photocurrent $I_{\text{hom}}(t)$ is therefore a classical quantity. It is given by

$$I_{\text{hom}}(t) = \bar{I}_{\text{hom}}(t) + \xi_{\text{hom}}(t). \quad (15)$$

where $\xi_{\text{hom}}(t)$ is local-oscillator shot noise, which in this theory is the only source of noise in the whole system. From Eqs.(4) and (14), the expected value $\bar{I}_{\text{hom}}(t)$ is

$$\bar{I}_{\text{hom}}(t) = \sqrt{\eta\varepsilon} \text{Tr}[\rho(t)\sigma_x] + \sqrt{\varepsilon} 2\beta\phi(t). \quad (16)$$

Here $\phi(t)$ is not set to its average value of zero because it is, in principle, known at any time as it is determined by the prior classical photocurrent via Eq. (8).

In the quantum trajectory theory, the photocurrent noise directly affects the atom, via a nonlinear stochastic term in the atom's master equation [12]

$$d\rho = dt\mathcal{D}[\sigma]\rho + \sqrt{\eta\varepsilon} dW_{\text{hom}}(t)\mathcal{H}[\sigma]\rho - idt[H_{\text{fb}}, \rho]. \quad (17)$$

The first term represents the usual damping, with

$$\mathcal{D}[A]B \equiv ABA^\dagger - \frac{1}{2}A^\dagger AB - \frac{1}{2}BA^\dagger A. \quad (18)$$

The second term represents the conditioning by the measurement, with $dW_{\text{hom}}(t) = \xi_{\text{hom}}(t)dt$ and

$$\mathcal{H}[A]B \equiv AB + BA^\dagger - \text{Tr}[AB + BA^\dagger]B. \quad (19)$$

The final Hamiltonian is due to the feedback. It is identical to the term due to feedback in the fundamental atomic Hamiltonian Eq. (13), namely

$$H_{\text{fb}}(t) = \sqrt{\eta}\beta\phi(t)\sigma_y. \quad (20)$$

Now consider the limit of instantaneous feedback on the atomic time-scale, $\tau \ll 1$. In this limit

$$2\beta\phi(t) = gI(t)/\sqrt{\varepsilon} \quad (21)$$

and thus we can derive

$$I_{\text{hom}}(t) = (1-g)^{-1} \{ \xi(t) + \sqrt{\eta\varepsilon} \text{Tr}[\rho(t)\sigma_x] \}. \quad (22)$$

Hence from Eqs. (20) and (21),

$$H_{\text{fb}}(t) = \lambda \frac{1}{2} \sigma_y \{ \text{Tr}[\rho(t)\sigma_x] + \xi(t)/\sqrt{\eta\varepsilon} \}, \quad (23)$$

where it is to be understood that t on the right-hand side of this equation actually stands for $t - 0^+$. The feedback parameter λ is given by

$$\lambda = g\eta/(1-g) \in (-\eta, \infty). \quad (24)$$

Thus far we still have a nonlinear stochastic equation for the conditioned atomic state matrix ρ , which is not easy to work with. However, in the Markovian limit it is possible to average over the stochasticity both in the measurement and feedback terms. As explained in Refs. [13,14], the result is the master equation

$$\dot{\rho} = \mathcal{D}[\sigma]\rho - i\lambda[\frac{1}{2}\sigma_y, \sigma\rho + \rho\sigma^\dagger] + \frac{\lambda^2}{\eta\varepsilon} \mathcal{D}[\frac{1}{2}\sigma_y]\rho. \quad (25)$$

This equation, and the following relation between λ and the in-loop squeezing

$$S_{\text{in}}^X(\bar{\omega}) = \frac{1 + g^2(\varepsilon^{-1} - 1)}{(1-g)^2} = 1 + \frac{2\lambda}{\eta} + \frac{\lambda^2}{\eta^2\varepsilon}. \quad (26)$$

are the central results of this work. In Eq. (26), we still have $\bar{\omega} \ll \tau^{-1}$, but now also $\bar{\omega} \gg 1$. This ensures that the atomic variables (with characteristic time scale of unity) do not contribute significantly to the spectrum at $\bar{\omega}$, so that Eq. (10) is still valid.

From the master equation Eq. (25) it is easy to derive the following dynamical equations:

$$\text{Tr}[\dot{\rho}\sigma_x] = -\gamma_x \text{Tr}[\rho\sigma_x] \quad (27)$$

$$\text{Tr}[\dot{\rho}\sigma_y] = -\gamma_y \text{Tr}[\rho\sigma_y] \quad (28)$$

$$\text{Tr}[\dot{\rho}\sigma_z] = -\gamma_z \text{Tr}[\rho\sigma_z] - C \quad (29)$$

Only the equation for σ_y is unaffected by the feedback, with $\gamma_y = 1/2$. The new decay rate for σ_x is

$$\gamma_x = \frac{1}{2} \left[1 + 2\lambda + \frac{\lambda^2}{\eta\varepsilon} \right], \quad (30)$$

and the modified parameters for σ_z are

$$\gamma_z = \gamma_y + \gamma_x, \quad C = 1 + \lambda, \quad (31)$$

In steady state $\text{Tr}[\rho_{\text{ss}}\sigma_x] = \text{Tr}[\rho_{\text{ss}}\sigma_y] = 0$ and $\text{Tr}[\rho_{\text{ss}}\sigma_z] = -1 + \lambda^2/[2\eta\varepsilon(1+\lambda) + \lambda^2]$.

The most interesting of these results is that negative feedback can reduce the decay rate of the x component of the atomic dipole below its natural value of $1/2$. From Eq. (26) it can be re-expressed as

$$\gamma_x = \frac{1}{2} [(1-\eta) + \eta S_{\text{in}}^X(\bar{\omega})]. \quad (32)$$

This clearly shows that γ_x has two contributions: $\frac{1}{2}(1-\eta)$ from the vacuum input and $\frac{1}{2}\eta S_{\text{in}}^X(\bar{\omega})$ from the in-loop squeezed light. The greatest reduction occurs for minimum in-loop fluctuations as in Eq. (11), for which

$$(\gamma_x)_{\text{min}} = \frac{1}{2} (1 - \eta\varepsilon), \quad \text{for } \lambda = -\eta\varepsilon. \quad (33)$$

The slower decay of σ_x can be directly observed in the power spectrum of the fluorescence of the atom into the vacuum modes. This measures the photon flux per unit frequency into these modes and is defined by

$$P(\omega) = \frac{1-\eta}{2\pi} \langle \tilde{\sigma}^\dagger(-\omega) \sigma(0) \rangle_{\text{ss}}. \quad (34)$$

From Eq. (25) this is easily evaluated to be

$$P(\omega) = \frac{(1-\eta)(\gamma_z - C)}{8\pi\gamma_z} \left[\frac{\gamma_x}{\gamma_x^2 + \omega^2} + \frac{\gamma_y}{\gamma_y^2 + \omega^2} \right]. \quad (35)$$

For the optimal squeezing ($\lambda = -\eta\varepsilon$) we have

$$P(\omega) = \frac{(1-\eta)\eta\varepsilon}{4\pi(2-\eta\varepsilon)} \left[\frac{1-\eta\varepsilon}{(1-\eta\varepsilon)^2 + 4\omega^2} + \frac{1}{1+4\omega^2} \right]. \quad (36)$$

This is plotted in Fig. 2 for $\eta = 0.8$ and $\varepsilon = 0.95$.

Comparison with Free Squeezing. To compare the above results with those produced by free squeezing we again assume that the mode-matching of the squeezed modes into the atom is η , and that the squeezing is broadband compared to the atom. Assuming also that the input light is in a minimum-uncertainty state for the X and Y quadratures [1], it can be characterized by a single real number L , with

$$S_{\text{in}}^X(\omega) = L = 1/S_{\text{in}}^Y(\omega). \quad (37)$$

In conventional notation [8], $L = 2N + 2M + 1$, where $M^2 = N(N+1)$. This yields the master equation

$$\dot{\rho} = (1-\eta)\mathcal{D}[\sigma]\rho + \frac{\eta}{4L}\mathcal{D}[(L+1)\sigma - (L-1)\sigma^\dagger]\rho, \quad (38)$$

which leads again to Eqs.(27)–(29), but with

$$\gamma_x = \frac{1}{2} [(1-\eta) + \eta L], \quad (39)$$

$$\gamma_y = \frac{1}{2} [(1-\eta) + \eta L^{-1}], \quad (40)$$

$$\gamma_z = \gamma_x + \gamma_y, \quad C = 1. \quad (41)$$

For $L < 1$ the decay of σ_x is again inhibited. The crucial observation to be made is that the dependence of γ_x on the degree of X quadrature squeezing of the input light is exactly the same as for in-loop squeezing, as is seen by comparing Eqs. (37) and (39) with Eq. (32). The only difference between the two cases is that C is unaffected by the free squeezing and that γ_y is not increased by the in-loop squeezing. The latter is a direct consequence of the fact that an in-loop field is not bound by the

usual two-time uncertainty relations. The free squeezing fluorescence spectrum is again given by Eq. (35). This is also plotted in Fig. 2 for $\eta = 0.8$ and $L = 0.05$. As this figure shows, the spectra are certainly not identical, but the sub-natural linewidth is much the same in both.

To conclude, line-narrowing of an atom is not a diagnostic of free squeezing. Rather, it requires only temporal anticorrelations of one quadrature of the input field (for times much shorter than the atomic lifetime) such as can be produced by a negative electro-optic feedback loop. The dependence of the line-narrowing on the input squeezing and the degree of mode-matching is the same for in-loop squeezing as for free squeezing. Because the quadrature operators of an in-loop field do not obey the usual two-time commutation relations, the reduction in noise in one quadrature does not imply an increase in noise in the other. Hence the line-narrowing of one quadrature of the atomic dipole by in-loop squeezing does not entail the line-broadening of the other quadrature. What significance this difference has in the physics of more complex atomic interactions with squeezed light [7] is a question requiring much investigation.

In-loop squeezing is generally easier to produce than free squeezing for a number of reasons. First, in-loop squeezing does not require expensive and delicate sources such as nonlinear crystals, but rather off-the-shelf electronic and electro-optical equipment. Second, the amount of squeezing is limited only by the efficiency of the photodetection. For homodyne detection, as required here, an efficiency of 95% is readily obtainable [15] and would enable in-loop squeezing of 95%. Third, in-loop squeezing can be produced at any frequency for which a coherent source is available, so experiments could be conducted on any atomic transition. The one difficulty with in-loop squeezing is that it requires a feedback loop response time much shorter than an atomic lifetime, but this would not be a problem for metastable transitions. Thus as well as giving us a better theoretical understanding of the effects on matter of light with fluctuations below the standard quantum limit, in-loop squeezing should be a practical alternative to free squeezing in the experimental investigation of these effects.

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FIG. 1. Diagram of the experimental configuration discussed. All beam splitters are 50:50. The atom is represented by the small ellipse at the focus of $b_{\text{in}}(t)$. The difference $I_{\text{hom}}(t)$ between the photocurrents at detectors D1 and D2 is amplified and split. The two signals (with opposite sign) are fed back to the two electro-optic modulators (EOM).

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FIG. 2. Plot of the Power Spectrum $P(\omega)$ of the fluorescence into the vacuum modes, for in-loop squeezing (solid) and free squeezing (dotted), with mode-matching $\eta = 0.8$ and squeezing $S_{\text{in}}^X(\bar{\omega}) = 0.05$. The linewidth for in-loop squeezing is slightly broader because the contribution from σ_y is not broadened in this case. The natural-width spectrum of a very weakly driven atom (dashed) is scaled up for comparison.

